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DESIGN OF A FILM-COOLED ENTRAINING DIFFUSER.(U)
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SUFFIELD TECHNICAL NOTE

NO. 444

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DESIGN OF A FILM-COOLED ENTRAINING DIFFUSER (U)

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by

S.B. Murray

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TABLE OF CONTENTS

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DEFENCE RESEARCH ESTABLISHMENT SUFFIELD
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SUFFIELD TECHNICAL NOTE NO. 444

DESIGN OF A FILM-COOLED ENTRAINING DIFFUSER (U)

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ABSTRACT

A film-cooled entraining diffuser is described which consists of a series of staged cylindrical rings, each overlapping the adjacent one so as to create annular slots for the entrainment of surrounding ambient air.

A two-step iterative design procedure is outlined. In step one an analysis similar to that first employed by von Karman is used to calculate the rate at which air is drawn into each slot. In step two these flow rates are used in a downstream-marching, iterative, implicit finite-difference method to calculate the development of wall jet boundary layers downstream of the slots.

Details about the design and manufacturing of a three-ring model structure are presented and proposed future experimental validation is discussed.

(U)

NOMENCLATURE

Symbols

A	is a cross-sectional area of flow.
C_p	is a specific heat at constant pressure.
D_1	is the diameter of the duct to which the film-cooled entraining diffuser is fastened.
D_2	is the diameter of the n-th ring element at the downstream end of the film-cooled entraining diffuser.
F	is a radiation shape factor.
i	is the number of an interior nozzle ring element.
K	is a loss coefficient associated with the entrance to an entrainment slot.
ℓ	is the length of a nozzle ring element, or ℓ is a mixing length in the turbulence model.
L	is the overall length of the film-cooled entraining diffuser.
n	is the number of nozzle ring elements comprising the film-cooled entraining diffuser.
p	is the static pressure in the boundary-layer calculations and is assumed to be a function of x alone.
P	is the uniform static pressure at the entrance or exit of a nozzle ring element.
Pr	is the molecular Prandtl number.
Pr_t	is the turbulent Prandtl number.
r	is the local radius in axisymmetric flow.
R	is the gas constant for primary and entrained streams.
T	is the fluid static temperature.
u	is the streamwise (x-direction) component of velocity.
v	is the transverse (y-direction) component of velocity.
w	is a slot width.
x	is the streamwise coordinate, measured parallel to the wall.
y	is the transverse coordinate, measured normal to the wall.
α_t	is the fluid eddy thermal conductivity.
δ	is the boundary-layer thickness.
κ	is von Karman's mixing-length constant.
λ	is a constant relating mixing length to boundary-layer thickness in the outer region of the boundary layer.

Nomenclature (continued)

μ	is the fluid dynamic viscosity.
ν_t	is the fluid eddy viscosity.
Π	is Coles' law of the wake profile parameter.
ρ	is the fluid static density.

Subscripts

e	denotes value in the free stream.
i	denotes value for an arbitrary nozzle ring element.
w	denotes value at the wall.
1	denotes value in the main stream at the entrance to a nozzle ring element.
2	denotes value in the entrained stream at the entrance to a nozzle ring element.
3	denotes value in the combined stream at the exit of a nozzle ring element.

Superscripts

k	is zero for two-dimensional flow and unity for axisymmetric flow.
'	denotes a time fluctuating quantity.

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1.0 INTRODUCTION

For over a decade now common use has been made of thrust augmenters to increase the propelling force of aircraft gas turbine engines, particularly at low speeds or in a static situation. In its simplest configuration a thrust augmenter takes the form of a constant diameter cylinder or nozzle which is positioned concentric to and slightly overlapping the exhaust duct as shown in Figure 1. The action of high viscous shear forces and turbulent entrainment act to draw surrounding ambient air through the overlap region and into the nozzle where it mixes in a turbulent fashion with the engine exhaust gas. Although the present study takes advantage of precisely this phenomenon, it does not concern itself with thrust augmentation, but rather with free entrainment of ambient air for the purpose of achieving a physically compact diffuser, efficient exhaust stream dispersion, and efficient wall film cooling, or any combination of these possibilities.

In an industrial application, for example, where maximum output of shaft power may be of most concern, it is beneficial to realize a pressure recovery by diffusing the turbine exhaust stream before discharging it to the atmosphere. Such a process could be accomplished by the use of a simple diffuser, but this is often impractical due to the small angle of divergence imposed on the constraining walls in order to prevent flow separation. A much higher effective rate of diffusion is possible with a slotted diffuser as shown experimentally by Frankfurt (1975) and others. This is so because the

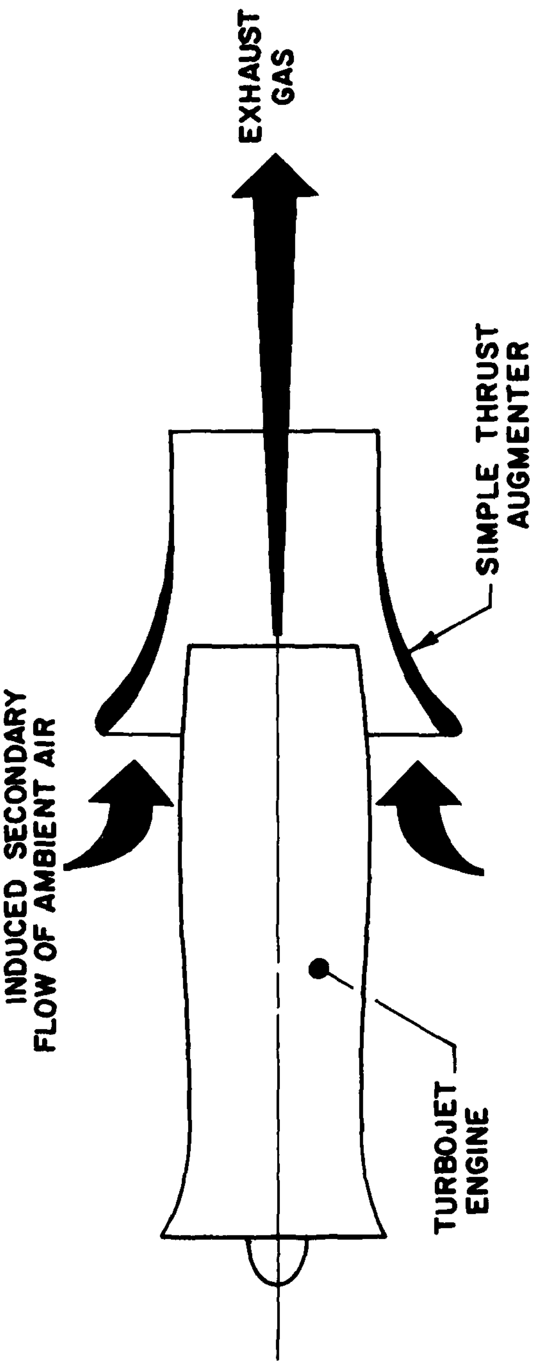

FIGURE 1 : A TURBOJET ENGINE AND SIMPLE THRUST AUGMENTER.

injected air streams form a series of energetic wall jet boundary layers which act to delay the onset of flow separation in a manner similar to that of slot blowing over the upper surface of airfoils.

In another conceivable application rapid cooling and dispersion of the exhaust plume may be necessary for reasons of safety or military strategy or to minimize the effects of local thermal pollution. In such a case one of the design objectives would be to incorporate relatively large slots in order to promote entrainment of the substantial volumes of ambient air needed to lower the bulk temperature of the combined stream at exit.

The objective in yet another application may be to isolate the diffuser walls from corrosive or hot exhaust gas either for reasons of military strategy or to prevent structural damage resulting from differential thermal expansion or surface pitting.

In any event it is evident that criteria governing the design of a film-cooled entraining diffuser will be dictated by its end use.

2.0 THE DESIGN PROCEDURE

Given the objectives of a particular diffuser its design must be carried out subject to certain geometric constraints. For example, it must be compatible with an exhaust duct of diameter D_1 and maintain an overall length of L and an exit diameter of D_2.

Inputs to the problem include parameters which characterize the gas flow within the duct, such as its mean velocity u_1 and absolute temperature T_1. If radiative heat-transfer processes are to be included in the analysis the effective gray-body emissivities of the duct material, diffuser material and exhaust gas must be specified. Ambient pressure and temperature are also required inputs.

Once the constraints and inputs have been supplied the "design procedure" includes determination of several more parameters including the number of nozzle ring elements or slots n, the length of each ring element ℓ_i, and the width of each slot w_i. These parameters are illustrated in Figure 2.

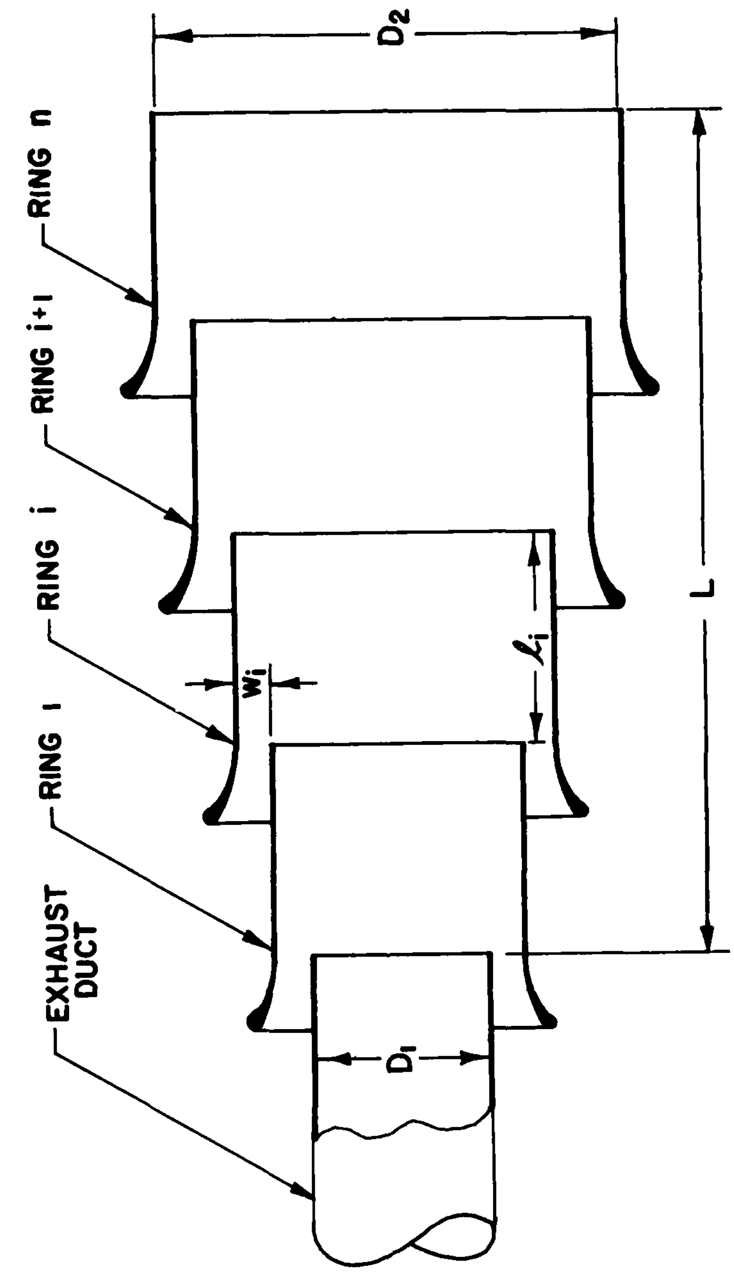

FIGURE 2 : A MULTIPLY-SLOTTED ENTRAINING DIFFUSER.

There are two major steps involved in the computational procedure. In Step 1 the rate of entrainment into each of the n slots is computed. Step 2 utilizes these flow rates in order to calculate the development of the wall jet boundary layer downstream of successive slots. Details of each step are laid out below.

2.1 Step 1: Calculating the Rates of Entrainment

As mentioned in the introductory remarks, it is possible to induce a secondary flow of ambient air by directing an exhaust jet into a nozzle or series of nozzle ring elements. Figure 3 illustrates how secondary air is drawn into the low-pressure throat region of a simple jet-nozzle combination under the influence of viscous shear forces and turbulent entrainment. Here it mixes with the exhaust gas to form a turbulent free shear layer. On leaving this zone the flow undergoes a pressure rise to ambient static pressure as it arrives at the exit plane of the nozzle.

The original calculations pertaining to this phenomenon were performed by von Karman (1949) with a view to increasing the thrust of turbojet engines. Although adequate information relating to thrust augmentation is available in the open literature there is an absence of data with regard to the optimization of entrainment systems from the points of view of plume and wall cooling. Consequently, in order to appreciate the potential of entrainment schemes, the original analysis on thrust augmentation has been repeated but with the inclusion of the energy equation and with particular emphasis on entrainment performance. The importance of heat-transfer effects is made clear by the theoretical analysis of Quinn (1976) who showed that increasing the temperature of the primary fluid degrades the performance of ejectors.

Briefly, the present analysis is as follows. Consider the simple jet-nozzle combination of Figure 3. The cross-sectional area of the nozzle is taken to be constant and equal to A_3 with pressure P_3 at the nozzle exit plane equal to the undisturbed ambient static pressure. The exhaust jet enters the nozzle with uniform velocity u_1 and temperature T_1 through area A_1. The secondary flow of cooling air is through area (A_3-A_1) and with uniform velocity u_2 and temperature T_2. It is assumed that the exiting flow is completely mixed and with monotonic velocity and temperature profiles of

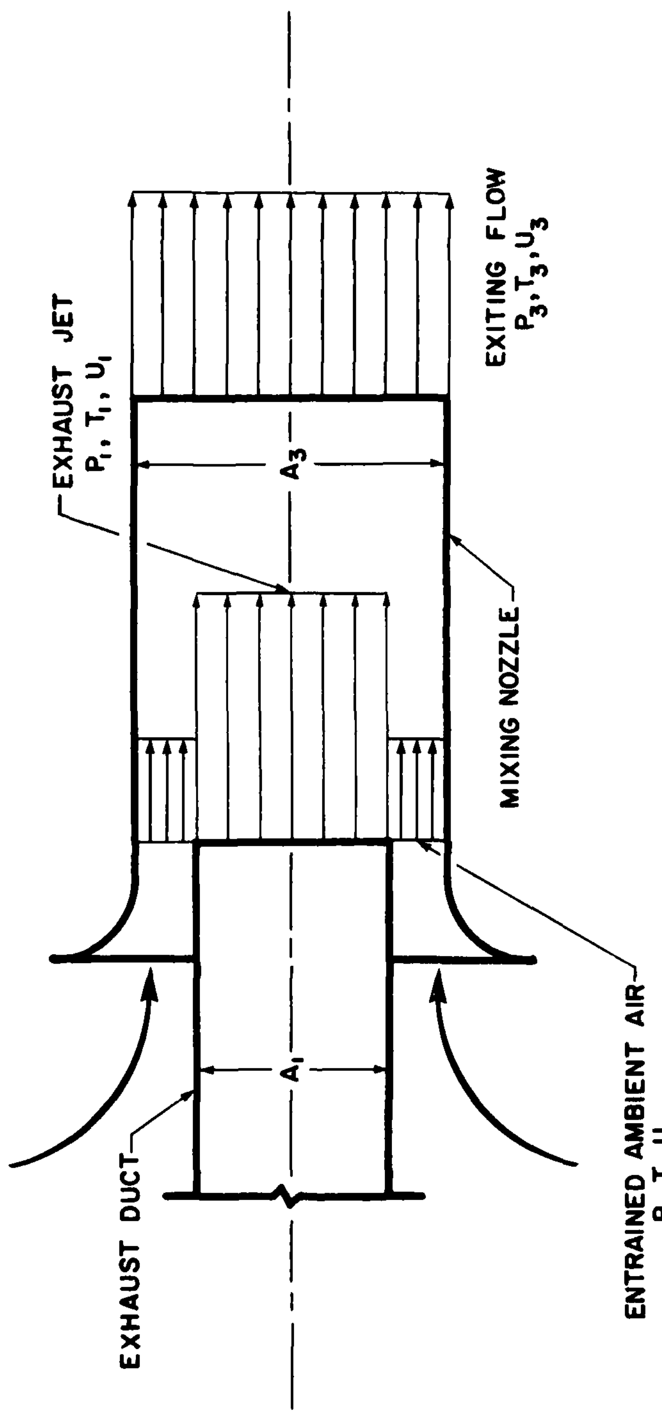

FIGURE 3 : ONE DIMENSIONAL ANALYSIS OF A SIMPLE JET-NOZZLE COMBINATION.

values u_3 and T_3, respectively. Furthermore, Bernoulli's equation can be written for the secondary flow between ambient infinity and a point in the throat just upstream of the mixing zone. Friction and turbulence losses in the nozzle are assumed to be negligible and the nozzle wall is considered to be adiabatic. The governing equations for incompressible mean flow in this system are:

Conservation of Mass

$$\rho_1 A_1 u_1 + \rho_2 (A_3 - A_1) u_2 = \rho_3 A_3 u_3 \tag{1}$$

Conservation of Momentum

$$P_2 A_3 - P_3 A_3 = \rho_1 A_1 u_1 (u_3 - u_1) + \rho_2 (A_3 - A_1) u_2 (u_3 - u_2) \tag{2}$$

Conservation of Energy

$$\rho_1 A_1 u_1 \left(C_{p_1} T_1 + \frac{u_1^2}{2}\right) + \rho_2 (A_3 - A_1) u_2 \left(C_{p_2} T_2 + \frac{u_2^2}{2}\right) = \rho_3 A_3 u_3 \left(C_{p_3} T_3 + \frac{u_3^2}{2}\right) \tag{3}$$

Bernoulli's Equation for the Entrained Stream

$$P_3 = P_2 + (1+K) \frac{\rho_2 u_2^2}{2} \tag{4}$$

Equation of State

$$P = \rho R T \tag{5}$$

where A is cross-sectional area of flow,
u is streamwise velocity,
T is static temperature,
P is static pressure,
ρ is static density,
K is an entrance loss coefficient associated with the secondary stream,
C_p is the specific heat at constant pressure, and
R is the gas constant.

The solution to this system of equations for the case where the primary gas is air and where the secondary stream suffers no entrance loss (K=0) is shown graphically in Figure 4. The ratio of jet area to nozzle area

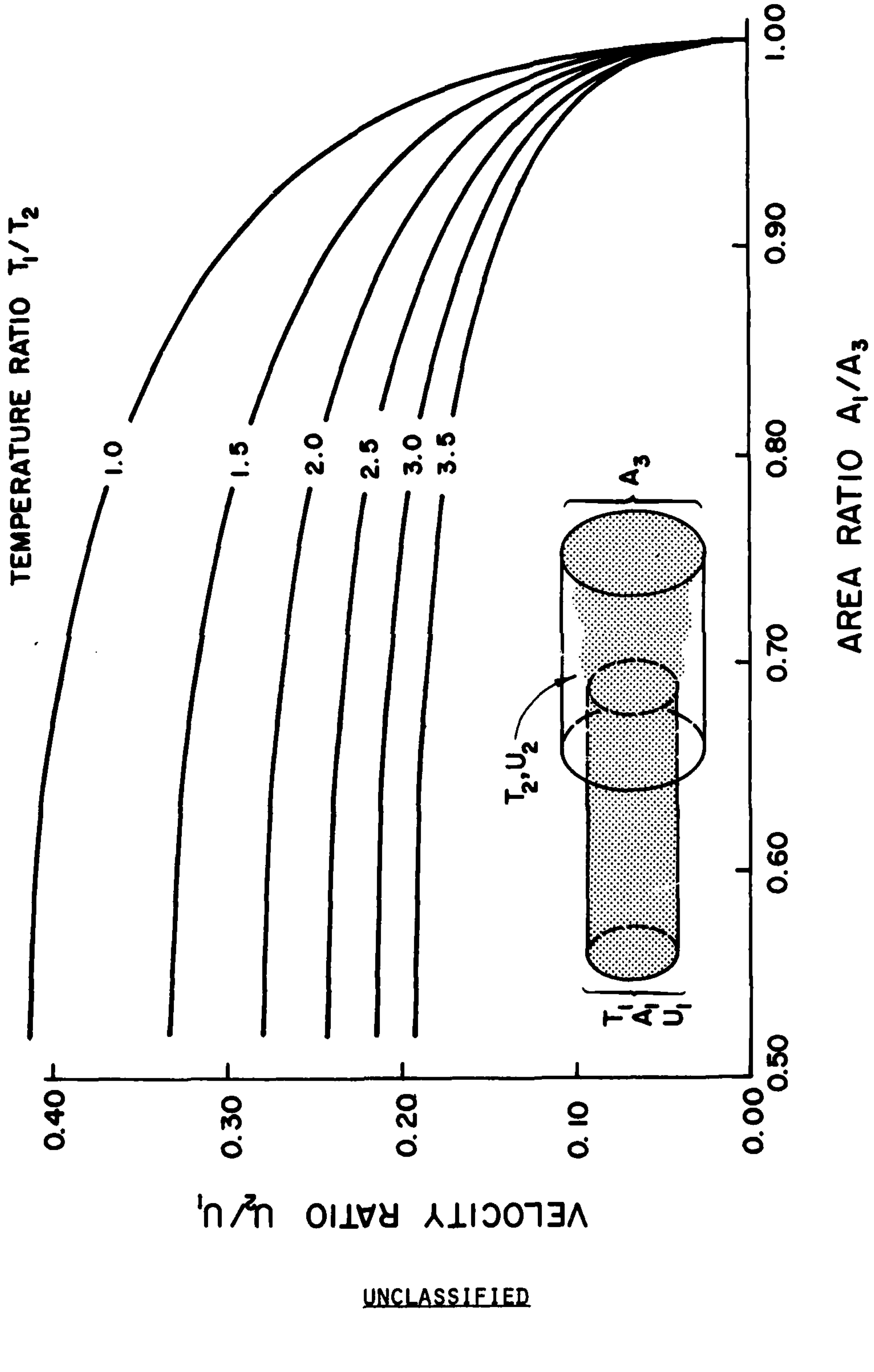

FIGURE 4 : PERFORMANCE OF THE SIMPLE JET-NOZZLE COMBINATION.

is plotted along the abscissa while the ratio of entrained air velocity to jet velocity is scaled along the ordinate. Calculations have been performed for several values of absolute temperature ratio, defined as the ratio of exhaust gas temperature to ambient air temperature. The curves indicate that the velocity of the entrained stream decreases for a given jet velocity as either the area ratio or the temperature ratio increases. Particular attention should be paid to the gradient of the curves at high values of area ratio. The steep slopes imply that the velocity of entrained air is very sensitive to slot width in this range and, since the cooling capability of the secondary stream is coupled to this velocity, one would anticipate a pronounced variation in wall cooling effectiveness as the slot width is varied. It should be emphasized that although these calculations yield ideal results McCormick (1969) has shown that, for the range of Reynolds numbers and large area ratios typical of the present application, excellent agreement between calculated and measured performance has been observed for a variety of thrust augmenters.

The unknown quantities in the system of equations above include the velocity of the entrained stream and the temperature and velocity of the mixed flow which exits the nozzle. Although the solution is relatively straightforward for the simple jet-nozzle combination of Figure 3, the matter is complicated somewhat when several nozzle ring elements are staged together to form a multiply-slotted assembly similar to that depicted in Figure 2. Under these conditions the static pressure at the exit plane of any interior ring element i is less than atmospheric pressure due to the presence of the downstream neighbouring ring element i+1. The actual drop in static pressure, however, is linked to the rate of entrainment into slot i+1, hence coupling the performance of adjacent ring elements. This dependency requires that the governing equations for each of the n ring elements be solved simultaneously, an undertaking of some magnitude due to the non-linearity of the equations.

An alternative to this method of solution and the one employed here is one of iteration. The basis of such an iterative procedure is as follows. The velocity of the entrained air stream in the first slot is guessed and

the system of equations governing mean flow in the first ring element is solved to yield the static pressure, static temperature and velocity of the mixed stream at the exit plane. These exit conditions become the entrance conditions for the second ring element and, since the static pressure at this location has just been specified, the velocity of the entrained stream in the second slot is fixed. This enables the governing equations for mean flow in the second ring element to be solved as they were for ring number one. This procedure is repeated for all n ring elements. A solution is realized if the calculated static pressure at the exit plane of the final ring element is equal to the undisturbed ambient static pressure, as it must for incompressible flow. A Newton root-finding technique is used to achieve convergence.

A computer program 'ENTRAIN' which calculates the rates of entrainment into each slot of an n-slot assembly appears in Appendix A. Up to twenty ring elements can be included in the analysis. The mean velocity and temperature of the main stream, the exhaust duct and ring radii, the entrance loss coefficients and the pressure and temperature of the reservoir from which each slot draws its air are the only required inputs. An example will be presented in Section 3.0.

2.2 Step 2: Calculating the Development of the Wall Jet Boundary Layers

Once the mean flow rate into each of the n slots has been determined by the procedure in Step 1 development of the wall jet boundary layers in the downstream direction can be readily computed. In the present study this is accomplished by numerical solution of the two-dimensional or axisymmetric turbulent boundary layer equations using a downstream-marching, iterative, implicit finite-difference method. Since full details of the model are given elsewhere by Murray (1979) only the highlights will be presented here for the sake of completeness.

The governing boundary-layer equations for incompressible turbulent flow in terms of time-averaged mean flow quantities are:

Continuity

$$\frac{\partial}{\partial x}(r^k \rho u) + \frac{\partial}{\partial y}(r^k \rho v) = 0 \qquad (6)$$

x-Momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{1}{r^k\rho}\frac{\partial}{\partial y}\quad r^k(\mu\frac{\partial u}{\partial y} - \rho\overline{u'v'}) \qquad (7)$$

Energy

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{r^k\rho}\frac{\partial}{\partial y}\quad r^k(\frac{\mu}{Pr}\frac{\partial T}{\partial y} - \rho\overline{T'v'}) \qquad (8)$$

State

$$p = \rho RT \qquad (5)$$

with the following Boussinesq eddy-diffusivity assumptions for the Reynolds stress and heat-transfer terms:

$$-\rho\overline{u'v'} = \rho\nu_t\frac{\partial u}{\partial y} \qquad (9)$$

$$-\rho\overline{T'v'} = \rho\alpha_t\frac{\partial T}{\partial y} \qquad (10)$$

where the definition of the turbulent Prandtl number is

$$Pr_t = \frac{\nu_t}{\alpha_t} \qquad (11)$$

Here ν_t and α_t are the eddy viscosity and eddy thermal conductivity.

The exponent k is equal to zero for plane flow and equal to unity for axisymmetric flow. The coordinate system is curvilinear in which x and y are distances along and normal to the body surface, with u and v the velocity components within the boundary layer in the x- and y- directions, respectively.

The boundary conditions associated with the above equations for the present application are:

Momentum

$$u(x,0) = 0$$
$$v(x,0) = 0 \qquad (12)$$
$$\lim_{y\to\infty} u(x,y) = u_e(x)$$

Energy

$$\frac{\partial T}{\partial y}(x,0) = \left.\frac{\partial T}{\partial y}\right|_w \qquad (13)$$

$$\lim_{y\to\infty} T(x,y) = T_e(x)$$

where subscripts w and e denote conditions at the wall and at the outer edge of the boundary layer, respectively. These equations fulfill the requirements of no slip at the wall as well as prescribing the streamwise distribution of wall heat flux. The outer edge velocity u_e and static temperature T_e are obtained from the inviscid flow calculation of Step 1 and must be consistent with the streamwise distribution of static pressure, p(x).

Before a solution to the system of equations defined by Equations 5 through 13 is possible, the form of the turbulent eddy viscosity must be specified. The shear stress in a conventional turbulent wall boundary layer is treated herein by the use of a two-layer inner-outer model based on the Prandtl mixing-length hypothesis. That is

$$\nu_t = \ell^2 \left|\frac{\partial u}{\partial y}\right| \qquad (14)$$

where the mixing length ℓ is given by Escudier (1965) as

$$\ell = \kappa y \text{ for } 0 \le y \le \frac{\lambda\delta}{\kappa} \text{ and}$$

$$\ell = \lambda\delta \text{ for } \frac{\lambda\delta}{\kappa} \le y \le \delta. \qquad (15)$$

Patankar and Spalding (1968) have recommended that the numerical constants be taken as $\kappa = 0.435$ and $\lambda = 0.09$. The edge of the boundary layer δ is defined as the point where the local velocity is equal to ninety-nine percent of that in the free stream.

In the near-wall region, where both laminar and turbulent components of the total shear stress are important, Van Driest's (1956) modification to the mixing length has been employed. The effects of pressure gradient and heat transfer, which are commonplace in the present application, have been accounted for in the mixing-length formula by Cebeci and Smith (1974).

This basic two-equation model for conventional turbulent wall boundary layers has been extended to handle the case of wall jets in the following manner. As long as the wall jet boundary layer exhibits a local jet maximum in its velocity profile the remnant of the main-stream boundary layer and the wall jet which constitute the combined layer are considered to be separate entities. Hence the two-equation model is applicable to each region independently. The eddy-viscosity profile is made continuous between eddy-viscosity maxima by fitting a cosine fairing between these maxima as performed by Dvorak (1973). This completes the set of closure assumptions that are necessary before a solution is possible.

The differential equations for the conservation of mass, momentum and energy are expressed in finite-difference form using three-point central differencing in the y-direction and three-point upstream differencing in the x-direction. In this manner each of the momentum and energy equations breaks down into a system of linear algebraic equations in tridiagonally-banded form that is solvable by rapid efficient means. Once the inflow boundary conditions are specified the remainder of the flow field is solved by marching in a downstream manner from the near-slot region to the end of the nozzle ring element.

2.3 Iteration

Unfortunately, as in the case of many coupled two-step solution procedures for aerodynamic problems, the design of a film-cooled entraining diffuser involves iteration. This is necessitated by the fact that flow development depends on factors such as pressure gradient and radiation shape factors. These quantities are, in part, functions of geometry, the details of which may not be known until the flow field is computed. In a case where a given degree of film-cooling protection is desired, for example, the length of nozzle ring element that is acceptable will not become apparent until the computations are carried out. Only then will it be seen where the ring temperature exceeds some prespecified upper limit. However, before such calculations can be performed the pressure gradient that is imposed on the boundary layer must be defined. This depends on the unknown ring length.

Consequently, it becomes necessary to guess at this unknown length and to check its validity a posteriori.

Other iterations may be necessary in the event that the sum of the computed ring lengths exceeds some overall limiting length. It would then be necessary either to alter the criterion which governs ring length or to adjust the slot width in an effort to accomplish the desired end result in a slightly different manner.

A flow chart which illustrates all possible iterations is shown in Figure 5. Depending on the objectives of the design, the constraints and the quality of first guesses, it may be possible to eliminate one or more of the loops shown.

3.0 SAMPLE DESIGN PROBLEM

In order to illustrate the design procedure a component of equipment recently designed at DRES, which incorporates a film-cooled entraining diffuser, will serve as an example. The details of this apparatus are unimportant as far as this exercise is concerned. Suffice it to say that the objectives of the design are

(a) to achieve a wall film-cooling efficiency in excess of 95 percent, and

(b) to accomplish significant exhaust stream diffusion subject to the following constraints:

   i) The film-cooled diffuser must be compatible with a duct of 28-inch diameter. This duct is of unspecified upstream configuration but can be considered a black body cooled to an efficiency of 95 percent.

   ii) The overall length of the structure must be held to 36 inches (measured in the axial direction from the first slot exit) with the exit diameter not greater than 33 inches due to restrictions on available space.

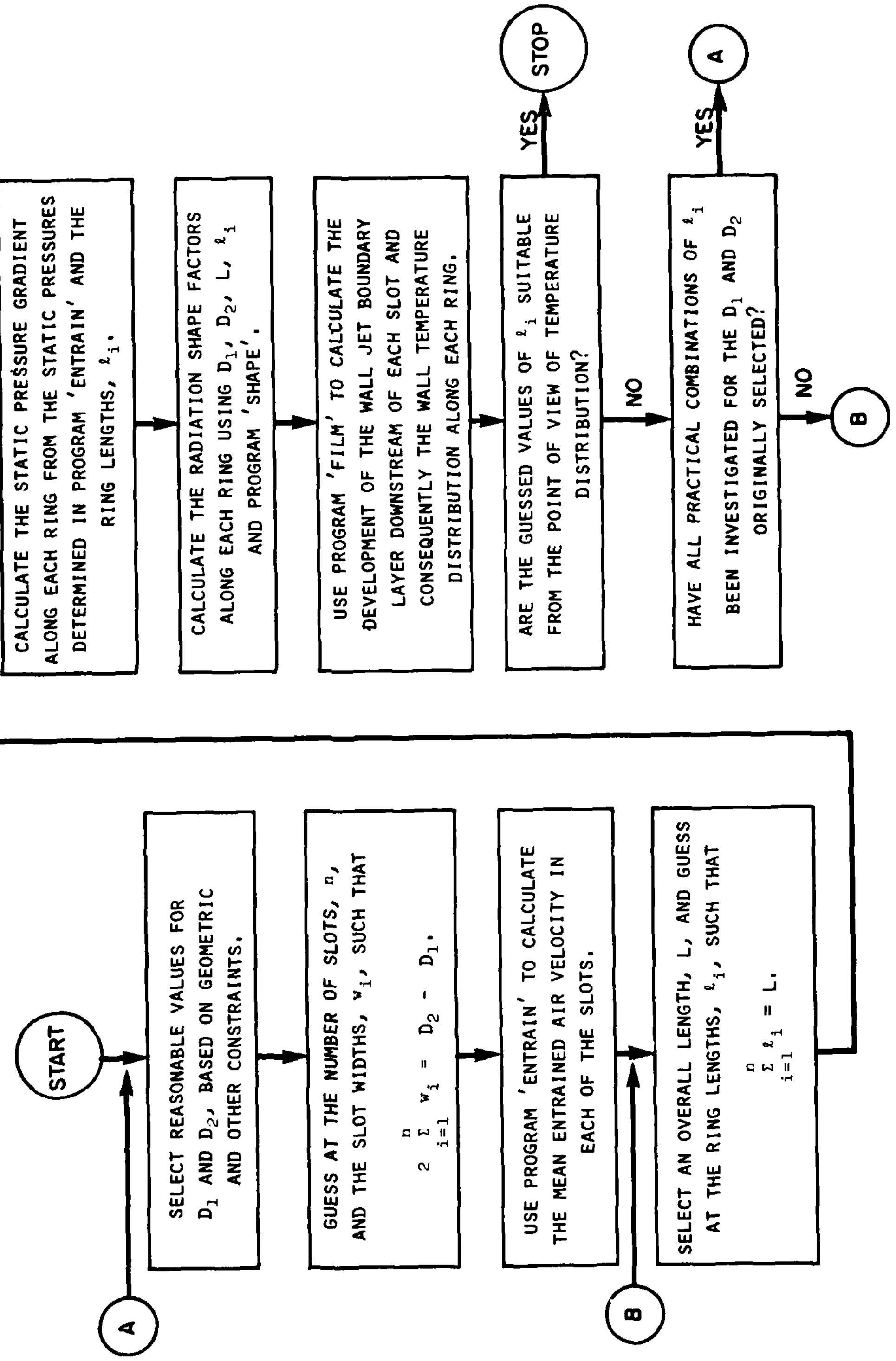

FIGURE 5 : FLOW CHART FOR THE DESIGN OF A MULTIPLY-SLOTTED ASSEMBLY.

iii) The film-cooled diffuser is to be constructed of 1/16-inch mild steel whose surface is treated to exhibit an emissivity of unity.

iv) The flow of hot gas is turbine exhaust at a uniform velocity of 151.9 fps, a temperature of 1048°R and an effective gray-body emissivity of 0.05.

v) Ambient pressure and temperature are 13.40 psia and 520°R, respectively.

Step 1: As an initial guess it will be assumed that the structure consists of three nozzle ring elements which overlap so as to create three entraining slots, each of 3/4-inch width. In combination with the ring material thickness this gives an overall outside diameter of 32-7/8 inches at the exit. Note that this dimension is just under the maximum allowable diameter of 33 inches. The resulting diffusion will therefore approach the maximum possible under these geometric constraints.

In order to calculate the rate of entrainment into each of the three slots an assumption must be made about entrance loss coefficients. These can be calculated from formulae found in any standard text on elementary fluid mechanics, such as that by John and Haberman (1971). For the geometry of the present application the loss coefficients for slots 1, 2 and 3 have been calculated to be 0.176, 0.150 and 0.150, respectively. These coefficients are typical of a well-rounded entrance with wiggle stripping located in the overlap region in order to maintain a constant slot spacing. The loss coefficient is somewhat higher for slot number one due to a 15-degree bend in the overlap region (as imposed by the apparatus upstream of the diffuser) and the presence of fastening clips which are required to secure the diffuser to the upstream duct.

Results of the entrainment calculation are presented in Figure 6. As shown, the mean velocities in slots 1, 2 and 3 are 42.5, 35.5 and 25.8 feet per second, respectively.

Step 2: Having computed these velocities it is now possible to calculate the flow development downstream of each slot. An initial guess must be

	MAINSTREAM PARAMETERS					ENTRAINED STREAM PARAMETERS				
SLOT NUMBER N	STATIC PRESSURE P (PSFA)	VELOCITY U1 (FPS)	TEMPERATURE T1 ('R)	AREA A1 (SQ.FT)		TOTAL PRESSURE PH (PSFA)	VELOCITY U2 (FPS)	TEMPERATURE T2 ('R)	AREA A2 (SQ.FT)	ENTRANCE LOSS K
ITERATION NUMBER 1										
1	1922.4381	151.9000	1048.0000	4.2761	*	1929.6000	75.9508	530.0000	0.4704	0.1760
2	1922.4497	144.3782	997.2292	4.7465	*	1929.6000	76.7424	530.0000	0.4950	0.1500
3	1922.4488	137.9959	953.1350	5.2414	*	1929.6000	76.7471	530.0000	0.5195	0.1500
EXIT	1922.4480	132.4764	914.9972	5.7609	*					
ITERATION NUMBER 2										
1	1927.1729	151.9000	1048.0000	4.2761	*	1929.6000	44.1597	530.0000	0.4704	0.1760
2	1927.8422	141.1946	1017.3462	4.7465	*	1929.6000	37.9969	530.0000	0.4950	0.1500
3	1928.5106	131.4227	992.5789	5.2414	*	1929.6000	29.9071	530.0000	0.5195	0.1500
EXIT	1929.2250	122.2404	973.9696	5.7609	*					
ITERATION NUMBER 3										
1	1927.3531	151.9000	1048.0000	4.2761	*	1929.6000	42.4869	530.0000	0.4704	0.1760
2	1928.0677	141.0265	1018.4534	4.7465	*	1929.6000	35.4738	530.0000	0.4950	0.1500
3	1928.7968	131.0293	995.1661	5.2414	*	1929.6000	25.6778	530.0000	0.5195	0.1500
EXIT	1929.6090	121.4966	978.9498	5.7609	*					
ITERATION NUMBER 4										
1	1927.3490	151.9000	1048.0000	4.2761	*	1929.6000	42.5253	530.0000	0.4704	0.1760
2	1928.0626	141.0303	1018.4279	4.7465	*	1929.6000	35.5329	530.0000	0.4950	0.1500
3	1928.7903	131.0384	995.1056	5.2414	*	1929.6000	25.7824	530.0000	0.5195	0.1500
EXIT	1929.5999	121.5145	978.8288	5.7609	*					

FIGURE 6 : RATES OF ENTRAINMENT INTO THE THREE-RING ASSEMBLY AS CALCULATED BY PROGRAM 'ENTRAIN'.

made at the ring lengths ℓ_1, ℓ_2 and ℓ_3. Experience has shown that the first ring is usually the shortest and that ring length increases to a maximum somewhere in the middle of the structure and then decreases toward the downstream end. This trend is the outcome of several factors:

i) Entrainment velocity decreases in the downstream direction due to a decelerating main stream and an increasing area ratio.

ii) Radiative heat transfer loads decrease as the distance from the exhaust duct increases.

iii) The build-up of cooling layers tends to improve cooling performance at downstream locations.

As a first guess the ring lengths ℓ_1, ℓ_2 and ℓ_3 will be 10, 14 and 12 inches, respectively, to give an overall length L of 36 inches as given in the list of constraints. The pressure gradient along each ring can now be calculated using the ring lengths above and static pressures in Figure 6. As well, now that the geometry has been specified, the distribution of radiation shape factor can be computed. A fairly simple approach to this calculation is outlined in Appendix B where a computer program 'SHAPE' is described. The results of 'SHAPE' for this example are summarized in Figure 7. Two shape factors are listed for each position along the ring structure at 1-inch axial intervals. One shape factor, F_{A-C}, relates to radiative heat transfer from the upstream duct to a ring element of infinitesimal width at the axial station in question. The other shape factor, F_{C-B}, relates to radiative heat transfer from this ring element to the atmosphere as viewed through the exit end of the diffuser assembly.

Before a boundary-layer calculation is possible assumptions must be made about the velocity profile in each of the slots and in the exhaust duct just upstream of the first slot. These assumptions must be made by the designer based either on his knowledge of the upstream flow or on experimental data. With the aid of the "law of the wall" and Coles' (1956) "law of the wake" realistic velocity profiles can be constructed once basic information about the boundary layers is provided.

RADIATION SHAPE FACTORS FOR A TAPERED DUCT

WITH CAPPED ENDS A AND B

AXIAL POSITION INCHES	RADIUS INCHES	SHAPE FACTOR F_{A-C}	SHAPE FACTOR F_{C-B}
1.00000	14.06250	0.46331	0.04836
2.00000	14.12500	0.42979	0.05147
3.00000	14.18750	0.39841	0.05482
4.00000	14.25000	0.36909	0.05842
5.00000	14.31250	0.34176	0.06229
6.00000	14.37500	0.31634	0.06646
7.00000	14.43750	0.29274	0.07095
8.00000	14.50000	0.27086	0.07578
9.00000	14.56250	0.25062	0.08099
10.00000	14.62500	0.23191	0.08659
11.00000	14.68750	0.21464	0.09263
12.00000	14.75000	0.19872	0.09914
13.00000	14.81250	0.18405	0.10614
14.00000	14.87500	0.17054	0.11368
15.00000	14.93750	0.15811	0.12180
16.00000	15.00000	0.14668	0.13053
17.00000	15.06250	0.13617	0.13992
18.00000	15.12500	0.12650	0.15002
19.00000	15.18750	0.11761	0.16086
20.00000	15.25000	0.10944	0.17249
21.00000	15.31250	0.10192	0.18496
22.00000	15.37500	0.09500	0.19831
23.00000	15.43750	0.08863	0.21259
24.00000	15.50000	0.08276	0.22785
25.00000	15.56250	0.07736	0.24412
26.00000	15.62500	0.07237	0.26146
27.00000	15.68750	0.06777	0.27988
28.00000	15.75000	0.06353	0.29944
29.00000	15.81250	0.05961	0.32016
30.00000	15.87500	0.05598	0.34206
31.00000	15.93750	0.05262	0.36516
32.00000	16.00000	0.04951	0.38949
33.00000	16.06250	0.04663	0.41504
34.00000	16.12500	0.04396	0.44182
35.00000	16.18750	0.04147	0.46982
36.00000	16.24994	0.03917	0.49900

FIGURE 7 : RADIATION SHAPE FACTORS FOR THE THREE-RING ASSEMBLY AS CALCULATED BY PROGRAM 'SHAPE'.

In addition to satisfying the above laws flow in the slot must meet two other criteria. Firstly, since the slot flow is one of low Reynolds number, Coles' profile parameter Π is a function of Reynolds number (based on the momentum deficit thickness). Secondly, the velocity profile must be constructed such that, when integrated across the slot, it yields the same mean velocity as calculated in the entrainment analysis of Step 1. These constraints necessitate iterative procedures in setting up the slot velocity profiles. Parameters used to create the velocity profiles in the present example are summarized in Table I below.

Flow	Maximum Streamwise Velocity u_e fps	Boundary-Layer Thickness δ inches	Coles' Profile Parameter Π
main stream	151.9	2.0	0.550
slot 1	48.5	0.25	0.141
slot 2	40.3	0.25	0.080
slot 3	29.3	0.25	0.000

TABLE I: Boundary-Layer Parameters used to Define the Main Stream and Slot Flows

Results of the boundary-layer calculation are shown in Figures 8 and 9. All distances, velocities and temperatures are in units of inches, fps and °R, respectively. The vertical scale on each plot has been exaggerated by a factor of two in relation to the horizontal scale. The finite grid used to represent the flow field consists of 36 equal-spaced axial stations and approximately 100 variable-spaced radial stations. About eight minutes of central processing time were required on an IBM 360 time-sharing system. Double precision (eight-byte variables) was required.

Figure 8 shows velocity profiles at every second axial station. For the low entrained air velocities which are typical of the present

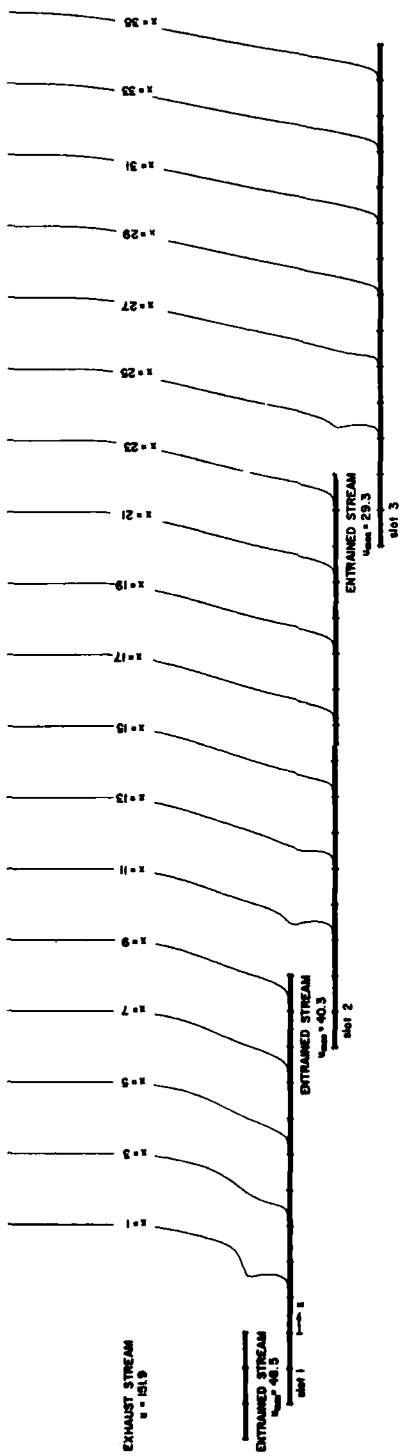

FIGURE 8 : VELOCITY PROFILES IN THE THREE-RING ASSEMBLY AS CALCULATED BY PROGRAM 'FILM'.

study the jets are seen to degenerate rather rapidly. This is more pronounced at downstream slots where the velocity of the entrained stream becomes progressively smaller.

Temperature profiles and isotherms are shown in Figure 9. The temperature profiles show clearly the degeneration of the zero temperature-gradient zone in the near-slot region. One important feature of the plot that is not evident due to the scale chosen is that temperature profiles near the slot exhibit a large gradient immediately adjacent to the wall. This is so because radiative heat transfer causes the wall to be at a higher temperature than the neighbouring fluid above it. This feature is more apparent on the isotherm plot in that the isotherm adjacent to the wall exhibits a discontinuity and change in direction.

Another observation of interest is that isotherms are mildly discontinuous in the near-slot region. These discontinuities occur because the eddy-viscosity model (responsible for describing the turbulent mixing process) changes once the local wall jet maximum in the velocity profile disappears.

4.0 EXPERIMENTAL VERIFICATION

In order to validate the design procedure presented in 2.0, it was decided to build and test the structure described in the example of 3.0. A detailed assembly drawing, shown in Figure 10, was prepared based on computed slot widths and ring lengths. A photograph of the manufactured structure appears in Figure 11. It is seen to consist of three overlapping mild-steel rings fastened together with steel wiggle-type stripping in order to maintain a 3/4-inch slot spacing. Metal tubing of circular cross section was machined and welded to the leading end of each ring so as to create smooth and well-rounded slot entries. The inner surface of each ring element was treated to achieve the highly-absorbing finish desired for the present application.

Although it is recognized that detailed experimental validation should include hot-wire anemometry and resistance-thermometry data in order to verify that velocity and temperature fields are as predicted, a much simpler

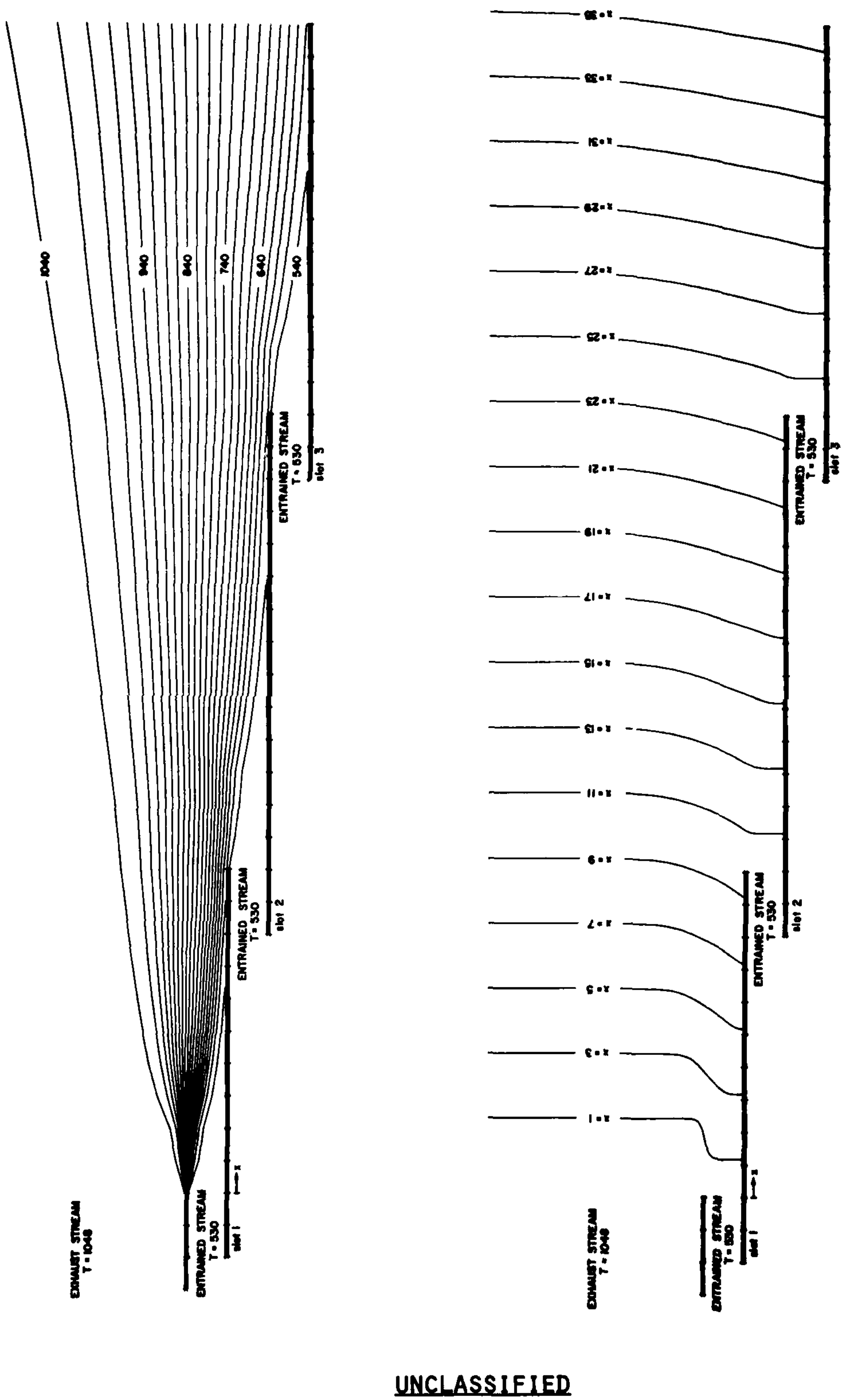

FIGURE 9 : ISOTHERMS AND TEMPERATURE PROFILES IN THE THREE-RING ASSEMBLY AS CALCULATED BY PROGRAM 'FILM'.

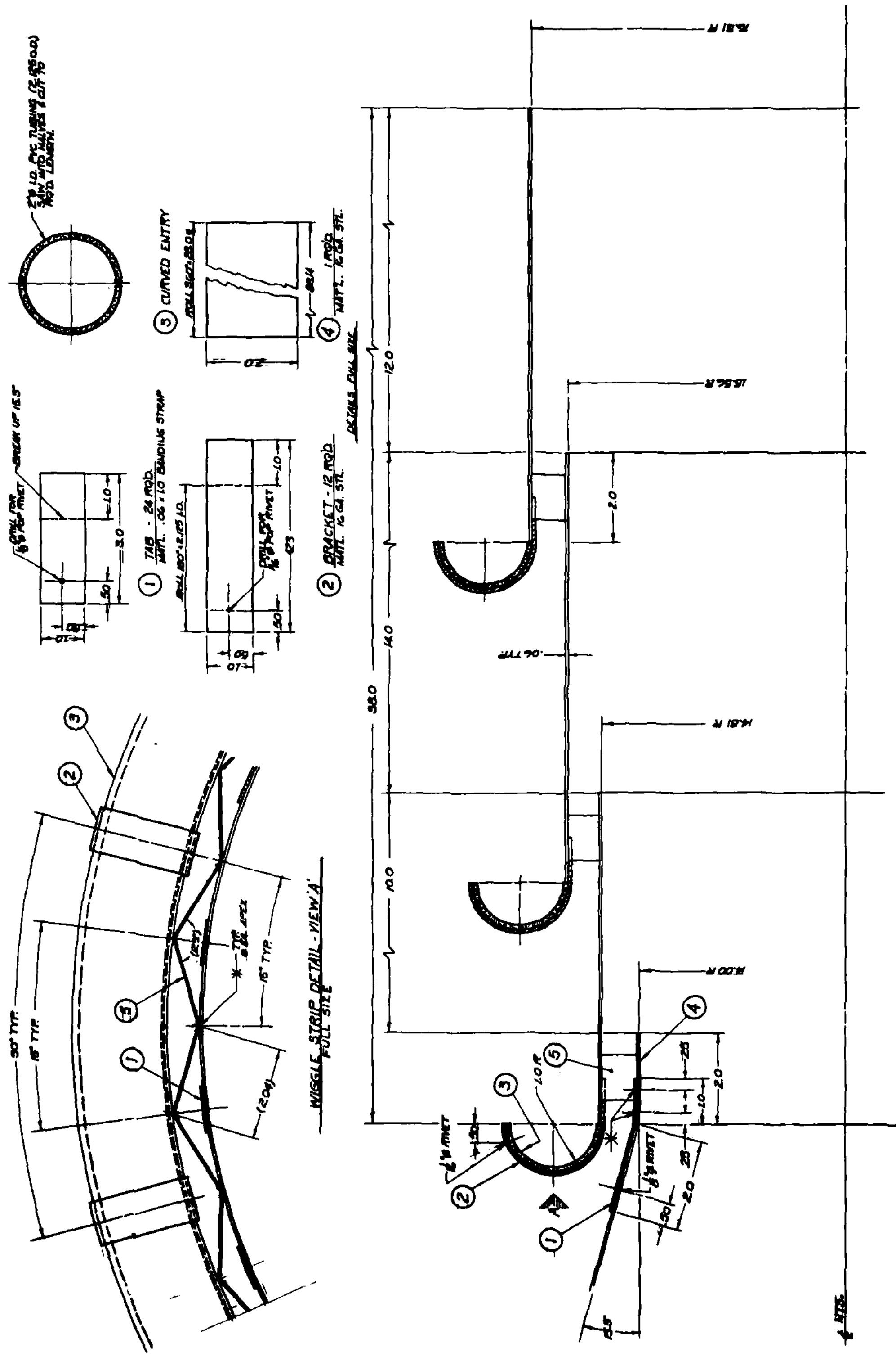

FIGURE 10 : ASSEMBLY DRAWING OF THE THREE-RING ASSEMBLY.

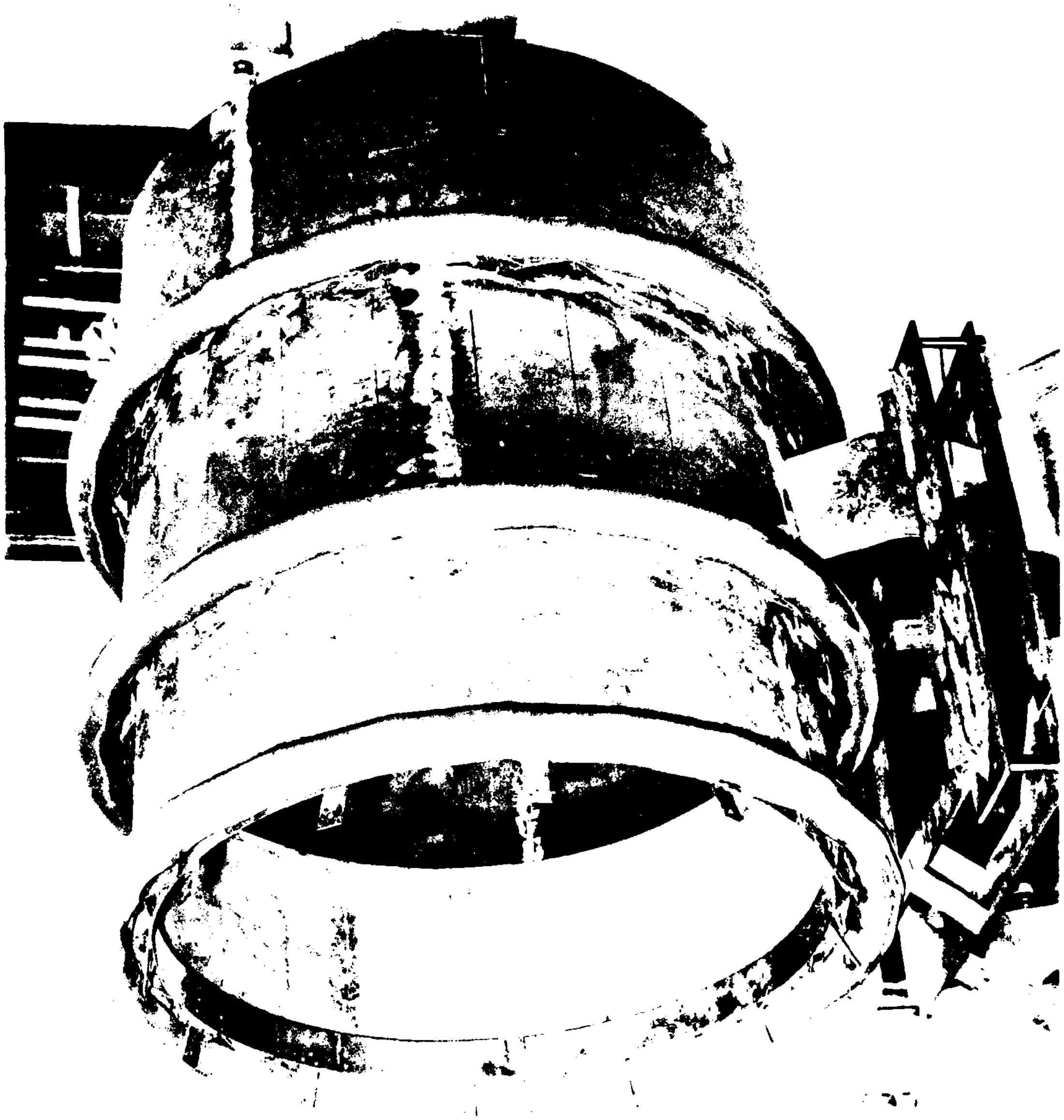

FIGURE 11 : ENTRAINING DIFFUSER WITH ROUNDED SLOT ENTRIES.

approach will be taken here due to limited time and resources. For the present, it is proposed that the design procedure be checked by two sets of simple measurements. In the first set stagnation probes and static pressure taps will be used to evaluate entrance loss coefficients and entrained air velocities. The second set of measurements will use thermocouples to measure skin temperature at various streamwise locations along the structure. If satisfactory results are not obtained it may be necessary to revert to more elaborate means of instrumentation.

It is intended to perform the experiments in the DRES film-cooling facility shown schematically in Figure 12. The facility consists of an Orenda gas turbine engine which supplies hot gas to a test section downstream of a flow control bypass vane.

5.0 CONCLUSIONS

An iterative design procedure for film-cooled entraining diffusers has been developed and results have been presented for a specific application. A test article based on this design has been fabricated and experiments with this apparatus will be conducted in the DRES film cooling facility. Results of these tests will be reported separately at a later date.

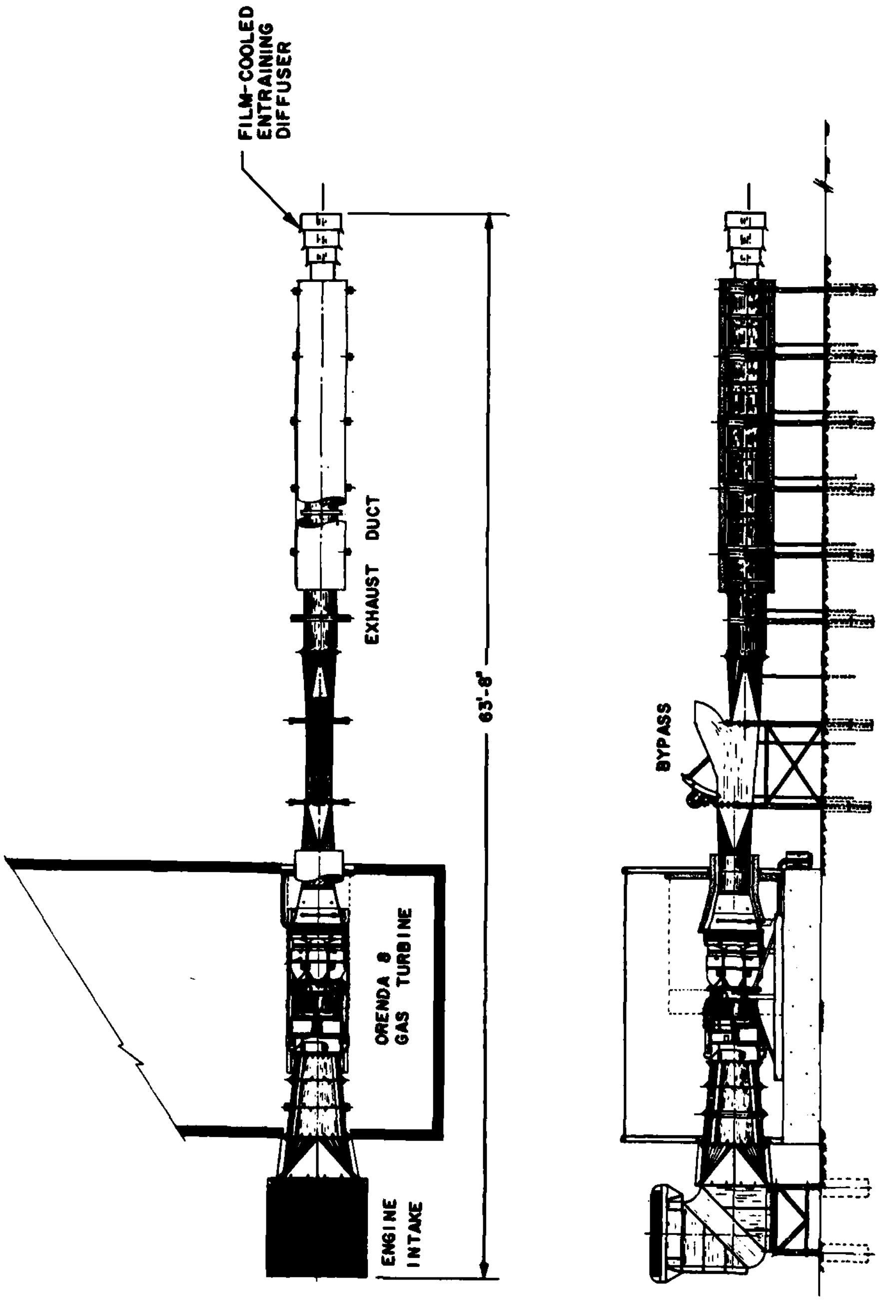

FIGURE 12 : THE DRES FILM-COOLING FACILITY.

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APPENDIX A: PROGRAM 'ENTRAIN'

The system of equations defined by Equations 1 through 5 can be rearranged so that the velocity, static pressure and temperature at the exit plane of any nozzle ring element are given by

$$u_3 = \frac{-\beta C_p + \sqrt{\beta^2 C_p{}^2 + 4\gamma\left(\frac{\alpha}{2g_c} - \delta C_p\right)}}{\left[\frac{\alpha}{g_c} - 2C_p\right]},$$

$$P_3 = \beta - u_3\delta, \text{ and}$$

$$T_3 = \frac{P_3 u_3}{\alpha}$$

where

$$\alpha = P_2\left[\frac{u_1}{T_1} \cdot \frac{A_1}{A_3} + \frac{u_2}{T_2} \cdot \left(1 - \frac{A_1}{A_3}\right)\right],$$

$$\beta = P_2\left[1 + \frac{u_1}{Rg_c} \cdot \frac{u_1 A_1}{T_1 A_3} + \frac{u_2}{Rg_c} \cdot \frac{u_2}{T_2} \cdot \left(1 - \frac{A_1}{A_3}\right)\right],$$

$$\delta = P_2\left[\frac{1}{Rg_c} \cdot \frac{u_1}{T_1} \cdot \frac{A_1}{A_3} + \frac{1}{Rg_c} \cdot \frac{u_2}{T_2} \cdot \left(1 - \frac{A_1}{A_3}\right)\right], \text{ and}$$

$$\gamma = P_2\left[C_p\left(u_1 \cdot \frac{A_1}{A_3} + u_2 \cdot \left(1 - \frac{A_1}{A_3}\right)\right) + \frac{u_1{}^2}{2g_c} \cdot \frac{u_1}{T_1} \cdot \frac{A_1}{A_3} + \frac{u_2{}^2}{2g_c} \cdot \frac{u_2}{T_2} \cdot \left(1 - \frac{A_1}{A_3}\right)\right].$$

The above method is used in the iterative solution method employed in program 'ENTRAIN'. A documented listing of the program appears in Figure A-1.

```
C**************************** ENTRAIN *********************************
C**************************** ENTRAIN *********************************
C**************************** ENTRAIN *********************************
C
C
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE THE STEADY STATE
C     RATES OF FLUID ENTRAINMENT INTO EACH SLOT OF A MULTIPY-SLOTTED
C     ENTRAINING DIFFUSER. THIS IS ACHIEVED BY SIMULTANEOUSLY SOLVING
C     THE EQUATIONS FOR THE CONSERVATION OF MASS, MOMENTUM AND ENERGY
C     FOR EACH RING IN THE ASSEMBLY. SINCE THE PERFORMANCE OF ANY ONE
C     NOZZLE RING ELEMENT IS DEPENDENT ON THE OTHERS THIS IS AN
C     ITERATIVE PROCEDURE.
C
C
C     UU1 AND UU2 ARE THE MEAN VELOCITIES OF THE MAIN AND ENTRAINED
C          STREAMS, RESPECTIVELY (FPS).
C
C     TT1 AND TT2 ARE THE STATIC TEMPERATURES OF THE MAIN AND ENTRAINED
C          STREAMS, RESPECTIVELY (°R).
C
C     PP2 IS THE STATIC PRESSURE AT THE THROAT WHERE THE MAIN AND
C          ENTRAINED STREAMS MEET (PSIA).
C
C     PPR IS THE TOTAL PRESSURE OF THE RESERVOIR FROM WHICH THE ENT-
C          RAINED FLUID IS BEING DRAWN (PSIA).
C
C     XLOSS IS THE LOSS COEFFICIENT FOR THE SLOT ENTRANCE AND INCLUDES
C          ALL LOSSES UP TO THE THROAT.
C
C     RR IS THE RING RADIUS (INCHES).
C
C     AA IS THE RING CROSS-SECTIONAL AREA (SQ.FT).
C
C     GAMMA IS THE RATIO OF SPECIFIC HEATS FOR BOTH GASES.
C
C     R IS THE GAS CONSTANT FOR BOTH GASES (FT-LBF/LBM-°R).
C
C     CP IS THE SPECIFIC HEAT AT CONSTANT PRESSURE FOR BOTH GASES
C          (FT-LBF/LBM-°R)
C
C
C     SUBSCRIPTS 1, 2 AND 3 REFER TO THE MAIN STREAM, ENTRAINED STREAM
C          AND EXITING MIXED STREAM, RESPECTIVELY.
C
C
      DIMENSION UU1(21),UU2(20),TT1(21),TT2(20),PP2(20),PPR(21),
     1 AA(21),XLOSS(21),RR(21)
      ROE(P,T)=P/R/T
C
C
C***************************** STEP 1 **********************************
C
C
C     INITIALIZATION SEGMENT.
C
      IN=5
      IOT=6
      GC=32.2
      R=53.35
      GAMMA=1.4
      CP=R*GAMMA/(GAMMA-1.0)
C
C
C     THE NUMBER OF SLOTS IS READ IN BELOW. IF THIS NUMBER EXCEEDS 20
C     THE JOB IS ABORTED.
C
      READ(IN,5)NSLOT
    5 FORMAT(I2)
      IF(NSLOT-20)20,20,10
   10 WRITE(IOT,15)
   15 FORMAT(///10X,'*** SPECIFIED NUMBER OF SLOTS IS TOO LARGE. JOB AB
     1ORTED.')
      STOP
   20 NSLT1=NSLOT+1
C
C
C     THE RING RADII (INCHES), RESERVOIR PRESSURES (PSIA) AND TEMPERA-
C     TURES (°R), AND THE ENTRANCE LOSS COEFFICIENTS ARE READ IN BELOW.
C
      READ(IN,25)(RR(J),J=1,NSLT1)
      READ(IN,25)(PPR(J),J=1,NSLT1)
      READ(IN,25)(TT2(J),J=1,NSLOT)
      READ(IN,25)(XLOSS(J),J=1,NSLOT)
   25 FORMAT(8F10.5)
C
C
C     THE DUCT AND SLOT AREAS ARE CALCULATED FROM THE RING RADII BELOW.
C
      DO 30 J=1,NSLT1
      AA(J)=3.1415926*RR(J)*RR(J)/144.
   30 PPR(J)=PPR(J)*144.
C
C
C     THE MAINSTREAM VELOCITY AND TEMPERATURE ARE READ IN BELOW.
C
      READ(IN,25)UU1(1),TT1(1)
C
C
C     HEADINGS ARE PRINTED BELOW.
C
      WRITE(IOT,35)
   35 FORMAT('1')
      WRITE(IOT,40)
   40 FORMAT(33X, 'MAINSTREAM PARAMETERS',25X ,'ENTRAINED STREAM'
     1 ,' PARAMETERS'//2X,'SLOT        STATIC',105X,'ENTRANCE')
      WRITE(IOT,45)
   45 FORMAT(1X,'NUMBER      PRESSURE       VELOCITY   TEMPERATURE      ARE
     1A A1',7X,
     2      'TOTAL PRESSURE  VELOCITY    TEMPERATURE     AREA A2         '
     3,'LOSS')
      WRITE(IOT,50)
   50 FORMAT(4X,'N         P (PSFA)       U1 (FPS)      T1 ('°R)        (SQ.F
     1T)',9X,
     2      'PR (PSFA)      U2 (FPS)       T2 ('°R)         (SQ.FT)          K
     3')
C
C
```

```
C***************************** STEP 2 **********************************
C
C
C     SOLVING FOR THE MEAN ENTRAINMENT VELOCITIES AND THE MEAN
C     MAINSTREAM TEMPERATURES AND VELOCITIES FOR EACH RING ELEMENT OF
C     A MULTIPLY-SLOTTED CONFIGURATION.
C
C
C     THE MAIN LOOP BEGINS. THE STATIC PRESSURE AT THE EXIT PLANE OF
C     THE FINAL RING ELEMENT IS A FUNCTION OF THE GUESSED ENTRAINMENT
C     V LOCITY IN THE FIRST SLOT. THIS PRESSURE MUST BE EQUAL TO AMB-
C     IENT STATIC PRESSURE FOR THE CORRECT SOLUTION. A NEWTON-RAPHSON
C     ROOT FINDING TECHNIQUE IS USED TO ACHIEVE CONVERGENCE. TWENTY
C     ITERATIONS ARE POSSIBLE.
C
      U2GES=0.50*UU1(1)
      DO 140 ITER=1,20
      WRITE(IOT,55)ITER
   55 FORMAT(//1X,'ITERATION NUMBER',I3)
C
C
C     THE SECONDARY LOOP BEGINS. SINCE THE CALCULATED STATIC PRESSURE
C     AT THE EXIT PLANE IS SOME UNSPECIFIED FUNCTION OF THE GUESSED
C     ENTRAINMENT VELOCITY IN THE FIRST SLOT, THE DERIVATIVE OF THE
C     FUNCTION (NEEDED FOR THE ROOT FINDING METHOD) IS CALCULATED BY
C     EVALUATING THE FUNCTION AT BOTH A HUNDREDTH OF A PERCENT ABOVE
C     AND BELOW THE GUESSED ENTRAINMENT VELOCITY.
C
      DO 135 JLOOP=1,2
      GO TO (60,65),JLOOP
   60 U2=1.00001*U2GES
      GO TO 70
   65 U2=0.99999*U2GES
   70 CONTINUE
C
C
C     THE TERTIARY LOOP BEGINS. THIS LOOP SOLVES THE EQUATIONS FOR THE
C     CONSERVATION OF MASS, MOMENTUM AND ENERGY FOR EACH OF THE RING
C     ELEMENTS. HENCE THE STATIC PRESSURE AT THE EXIT PLANE OF THE
C     FINAL RING ELEMENT IS OBTAINED AS A FUNCTION OF THE ENTRAINMENT
C     VELOCITY IN THE FIRST SLOT.
C
      DO 115 J=1,NSLOT
      U1=UU1(J)
      T1=TT1(J)
      T2=TT2(J)
      PR=PPR(J)
      A1=AA(J)
      A3=AA(J+1)
      ARAT=A1/A3
      IF(J-1)75,75,80
   75 P2=PR/(1.+(1.+XLOSS(J))*U2*U2/2./R/GC/T2)
      PP2(J)=P2
      GO TO 85
   80 P2=PP2(J)
      U2=SQRT((PR/P2-1.)*R*2.*GC*T2/(1.+XLOSS(J)))
   85 CONTINUE
      TERM1=U1/T1*ARAT
      TERM2=U2/T2*(1.-ARAT)
      ALPHA=P2*(TERM1+TERM2)
      TERM1=TERM1/R/GC
      TERM2=TERM2/R/GC
      DELTA=P2*(TERM1+TERM2)
      BETA=P2*(1.+U1*TERM1+U2*TERM2)
      TERM1=TERM1*R/2.
      TERM2=TERM2*R/2.
      GAMA=P2*(CP*(U1*ARAT+U2*(1.-ARAT))+U1*U1*TERM1+U2*U2*TERM2)
      A=ALPHA/2./GC-DELTA*CP
      B=BETA*CP
      C=-GAMA
      RADIC=B*B-4.*A*C
      IF(RADIC)90,100,100
   90 WRITE(IOT,95)
   95 FORMAT(///10X,'***  NEGATIVE RADICAL. JOB ABORTED.')
      STOP
  100 U3=(-B+SQRT(RADIC))/2./A
      P3=BETA-U3*DELTA
      T3=P3*U3/ALPHA
      UU1(J+1)=U3
      PP2(J+1)=P3
      TT1(J+1)=T3
      UU2(J)=U2
      GO TO (105,115),JLOOP
  105 A2=A3-A1
      WRITE(IOT,110)J,P2,U1,T1,A1,PR,U2,T2,A2,XLOSS(J)
  110 FORMAT(/3X,I2,3X,4(4X,F9.4),4X,'*',5(4X,F9.4))
  115 CONTINUE
C
C
C     THE TERTIARY LOOP ENDS.
C
C
C     P3PLS AND P3MIN ARE THE STATIC PRESSURES AT THE EXIT PLANE OF THE
C     FINAL RING ELEMENT, EVALUATED AT A HUNDREDTH OF A PERCENT ABOVE
C     AND BELOW THE GUESSED ENTRAINMENT VELOCITY IN THE FIRST SLOT,
C     RESPECTIVELY.
C
      GO TO (120,130),JLOOP
  120 P3PLS=P3
      WRITE(IOT,125)P3,U3,T3,A3
  125 FORMAT(/2X,'EXIT',2X,4(4X,F9.4),4X,'*')
      GO TO 135
  130 P3MIN=P3
  135 CONTINUE
C
C
C     THE SECONDARY LOOP ENDS.
C
C
C     THE NEWTON-RAPHSON ROOT FINDING TECHNIQUE IS USED TO EVALUATE THE
C     NEXT GUESS AT ENTRAINMENT VELOCITY IN THE FIRST SLOT.
C
      F=(P3PLS+P3MIN)/2.-PPR(NSLT1)
      FP=(P3PLS-P3MIN)/0.00002/U2GES
      U2OLD=U2GES
      U2GES=U2GES-F/FP
      IF(ABS((U2GES-U2OLD)/U2GES)-0.0001)145,145,140
  140 CONTINUE
  145 CONTINUE
C
C
C     THE MAIN LOOP ENDS.
C
C
      STOP
      END
```

FIGURE A-1: A DOCUMENTED LISTING OF PROGRAM 'ENTRAIN'.

APPENDIX B: PROGRAM 'SHAPE'

Two modes of radiative heat transfer are present in the diffuser: radiation between the diffuser walls and sources both upstream and downstream of the diffuser assembly, and radiation from the hot exhaust gas to the walls of the diffuser assembly. The combined effect of these modes of heat transfer is considered to be the simple additive result. This approach is justified since the gas is largely transparent.

Consider the first mode of radiation described above. The diffuser is of relatively complex geometry with regard to the calculation of radiation shape factors. However, since one of the goals of the present application is to ensure a relatively constant wall temperature via the use of air film cooling, radiative heat transfer between neighbouring regions within the diffuser is small. Clearly, the significant exchange of radiant energy is between the walls of the diffuser and both the upstream duct and downstream ambient surroundings. With this in mind the calculation of shape factors is simplified considerably. Although the diffuser consists of a series of staged cylindrical rings, for the purpose of computing these factors, it will be approximated by a conical surface whose cross-sectional radius varies linearly in the axial direction from that of the exhaust duct to that of the largest cylindrical ring as illustrated in Figure B-1. The most significant errors incurred by this approximation are for the near-slot region where the step created by the slot tends to shield the wall to a higher degree than this approach would indicate. Since the amount of taper is small these errors are considered to be negligible.

From the point of view of radiation the upstream duct is well represented by a disk of the same radius, temperature and emissivity situated at the entrance of the diffuser. Likewise, a perfectly absorbing disk at ambient temperature located at the exit plane simulates the cool ambient surrounding. These disks are labelled A and B respectively in Figure B-1.

The shape factor from disk A to an arbitrary ring element C of length 2dx is simply the difference between the shape factor from disks A to

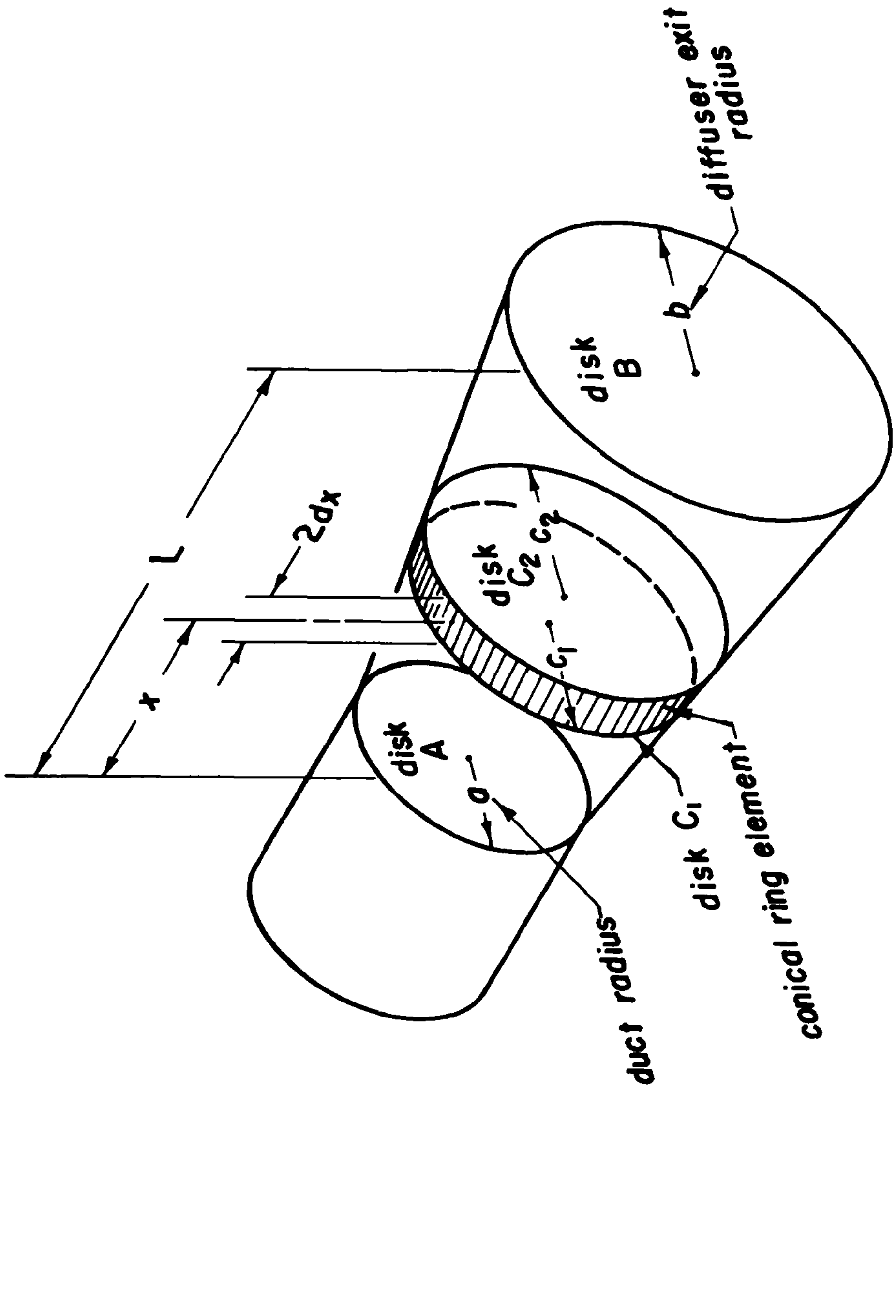

FIGURE B-1: SIMPLIFIED DIFFUSER FOR THE PURPOSE OF CALCULATING RADIATION SHAPE FACTORS.

C_1, and that from disks A to C_2. According to McAdams (1942) the shape factor from disk A to disk C_i, centred on the same axis a distance x apart, is

$$F_{A-C_i} = \frac{1}{2a^2}\left[x^2 + a^2 + c_i^2 - \sqrt{(x^2 + a^2 + c_i^2)^2 - 4a^2c_i^2}\right]$$

where a and c_i are the radii of disks A and C_i, respectively. The heat transfer per unit area of the conical ring element described above is then

$$q_{A-C} = \sigma\varepsilon_A\varepsilon_C\,(T_A^4 - T_C^4)\,\frac{A_A}{A_C}\,F_{A-C}$$

where $\frac{A_A}{A_C} F_{A-C} = \frac{\pi a^2}{2\pi(c_1+c_2)dx} \cdot \frac{1}{2a^2}\Big[[(x - dx)^2 + a^2 + c_1^2$

$-\sqrt{((x - dx)^2 + a^2 + c_1^2)^2 - 4a^2c_1^2}] - [(x + dx)^2 + a^2 + c_2^2$

$-\sqrt{((x + dx)^2 + a^2 + c_2^2)^2 - 4a^2c_2^2}]\Big]$.

Here ε is gray-body emissivity and T is absolute temperature. Taking the limit of the above expression as dx tends to zero, with the aid of L'Hopital's rule, yields

$$q_{A-C} = \sigma\varepsilon_A\varepsilon_C(T_A^4 - T_C^4)\quad\frac{x}{2c}\left[\frac{x^2 + a^2 + c^2}{\sqrt{(x^2 + a^2 + c^2)^2 - 4a^2c^2}} - 1\right]$$

where σ is Boltzmann's constant.

A similar analysis for the radiative heat transfer from the ring element to disk B gives

$$q_{C-B} = \sigma\varepsilon_B\varepsilon_C\,(T_C^4 - T_B^4)\;\frac{(L - x)}{2c}\left[\frac{(L - x)^2 + b^2 + c^2}{\sqrt{((L - x)^2 + b^2 + c^2)^2 - 4b^2c^2}} - 1\right].$$

The second mode of radiative heat transfer, namely that from the hot exhaust gas to the diffuser walls, is calculated by selecting a suitable effective gray-body emissivity for the hot gas and by assuming a shape factor near unity since the column of gas almost completely fills the interior of the diffuser. Effective gray-body emissivity for industrial and power plant exhaust gases having various concentrations of water vapour, carbon dioxide

and carbon particulates is given by Holman (1976).

A simple computer program 'SHAPE' which calculates the area-normalized shape factors above appears in Figure B-2.

REFERENCES

Holman, J.P., "Heat Transfer", McGraw-Hill Inc., New York, 1976.

McAdams, W.H., "Heat Transmission", McGraw-Hill Inc., New York, 1942.

```
C****************************** SHAPE **********************************
C****************************** SHAPE **********************************
C****************************** SHAPE **********************************
C
C
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE RADIATION SHAPE
C     FACTORS BETWEEN VARIOUS SURFACES IN A CONICAL DUCT WITH CAPPED
C     ENDS. IN PARTICULAR THE FOLLOWING SHAPE FACTORS ARE DESIRED,
C
C          1) THAT BETWEEN THE SMALL CAP, A, AND A RING ELEMENT, C, OF
C             INFINITESIMAL WIDTH, AND
C
C          2) THAT BETWEEN THE RING ELEMENT DESCRIBED ABOVE AND THE
C             LARGE CAP, B.
C
C     A IS THE RADIUS OF THE SMALL CAP (INCHES).
C
C     B IS THE RADIUS OF THE LARGE CAP (INCHES).
C
C     XL IS THE AXIAL LENGTH OF THE CONICAL DUCT (INCHES). '
C
C     DX IS THE AXIAL DISTANCE BETWEEN POINTS FOR WHICH THE SHAPE
C          FACTOR IS DESIRED (INCHES).
C
C
C     INITIALIZATION AND DATA INPUT.
C
      IOT=6
      IN=5
      READ(IN,10)A,B,XL,DX
   10 FORMAT(4F10.5)
      KOUNT=1.00001*(XL/DX)
      WRITE(IOT,20)
   20 FORMAT(//////41X,'RADIATION SHAPE FACTORS FOR A TAPERED DUCT'//
     1 50X,'WITH CAPPED ENDS A AND B'////
     2 45X,'AXIAL     RADIUS      SHAPE      SHAPE'/
     3 43X,'POSITION',14X,'FACTOR     FACTOR'//66X,'F          F'
     4 /44X,'INCHES     INCHES        A-C        C-B'//)
C
C
C     THE CALCULATIONS FOR SHAPE FACTOR BEGIN.
C
      DO 90 J=1,KOUNT
      IF(KOUNT-J)30,30,40
   30 X=XL-0.001*DX
      GO TO 50
   40 X=J*DX
   50 C=A+X/XL*(B-A)
C
C
C     THE SHAPE FACTOR FROM A TO C IS COMPUTED BELOW.
C
      PART1=X*X+A*A+C*C
      PART2=SQRT(PART1*PART1-4.*A*A*C*C)
      SHPAC=X/2./C*(PART1/PART2-1.0)
C
C
C     THE SHAPE FACTOR FROM C TO B IS COMPUTED BELOW.
C
      PART3=(XL-X)*(XL-X)+B*B+C*C
      PART4=SQRT(PART3*PART3-4.*B*B*C*C)
      SHPCB=(XL-X)/2./C*(PART3/PART4-1.0)
C
C
C     RESULTS ARE PRINTED BELOW.
C
      IF(KOUNT-J)60,60,70
   60 X=XL
   70 WRITE(IOT,80)X,C,SHPAC,SHPCB
   80 FORMAT(41X,4F10.5)
   90 CONTINUE
C
C
      STOP
      END
```

FIGURE B-2: A DOCUMENTED LISTING OF PROGRAM 'SHAPE'.

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13. ABSTRACT

A film-cooled entraining diffuser is described which consists of a series of staged cylindrical rings, each overlapping the adjacent one so as to create annular slots for the entrainment of surrounding ambient air.

A two-step iterative design procedure is outlined. In step one an analysis similar to that first employed by von Karman is used to calculate the rate at which air is drawn into each slot. In step two these flow rates are used in a downstream-marching, iterative, implicit finite-difference method to calculate the development of wall jet boundary layers downstream of the slots.

Details about the design and manufacturing of a three-ring model structure are presented and proposed future experimental validation is discussed.

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Turbulent Boundary Layers

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